



**MATHEMATICAL MODELS OF HUMAN  
CONTROL, CLASSIFICATION AND  
APPLICATION**

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In this paper we consider a general mathematical model for interactive software. We carry out a survey of existing and conceivable software, including one proposed and developed with author's participation. The paper highlights the objectives and specifics of various software including "independent presentation" of an object.

Keywords: human control, mathematical model, computer model, classification, application, training, learning

Рассмотрена общая математическая модель для интерактивного программного обеспечения. Проведен обзор существующих и возможных видов программного обеспечения, в том числе предложенного и разработанного авторами. В статье подчеркиваются цели и особенности различных видов программного обеспечения, в том числе "независимое представление" объекта.

Ключевые слова: *человеческое управление, математическая модель, компьютерная модель, классификация, приложение, тренировка, обучение*

# I. Introduction

Training devices for hunting, horsemanship and war are known since ancient times. Mechanical flight simulators appeared together with the development of aviation. Computers gave the opportunity to create simulations with real-time feedback and elements of virtual reality. These ideas were also implemented in computer games. Educational computer software and educational games developed together with the development of personal computers.

The paper contains a general mathematical model of such software, remarks on its implementation as a computer model, a list of known and possible kinds of software (some of which has been implemented with authors' participation).

## 2. Mathematical model for human control

$N_0 := \{0, 1, 2, \dots\}$  contains values of discrete time  $t$ ;  
 $R_+ := [0, \infty)$ ;

Denote  $X$  as the space of states  $x$  (including virtual media and objects in it);  $X_0 \subset X$  as the set of targets;  $Q: X \rightarrow R_+$  as the target function to be minimized;

$V$  as the set of observable (affectable by human interaction) elements of  $X$ ;  $W: X \rightarrow V$  is a given function;

$P$  as the set of random elements  $p$ ;

$U$  as the set of possible actions  $u$  by the user (control).

We will consider discrete models. Continuous models are obtained from discrete ones by setting time divisions/steps to zero.

We propose the system

$x[0]$  ( $x[0]=Z(p[0])$ ) is given (either  $x[0] \notin X_0$  or  $Q(x[0]) > 0$ ); (1)

$v[t]=V(x[t]); x[t+1]=F(x[t], u[t], p[t])$ , or  
 $x[t+1]=x[t]+G(x[t], u[t], p[t])$ ,  $t \in N_0$  (2)

where  $u[t]$  is the action of the user influenced by information  $v[t]$ ;  $Z(p):P \rightarrow X$  is a function of random generation of initial data.

The goal is either to reach  $x[t+1] \in X_0$  in minimal time or to minimize  $Q(x[t+1])$  in a given time.

Two options of input  $*x[0]$  by the user\* and  $*random*$  give us two modes: learning mode and exam mode.

In advanced software *TaskLang* [6] the user can choose functions  $F$  (or  $G$ ) too.

It may be  $x = \{x_1, \dots, x_n\}$ ,  $x_1, \dots, x_n$  are input independently; it is a necessary-collective task for  $n$  users.

The principle of duality [1]: (narrow  $V$  and wide  $U$ ) and (wide  $V$  and narrow  $U$ ) yield similar efficiency.

This principle extends for different kinds of human activity: Duality of available information and available goal achievement capacity.

### 3. Computer model specifics


#### 3.1. Input of information $v[t]$

- common (by means of eyesight, hearing, vibration - vestibular apparatus);
- by means of special devices (earphones, binocular displays);
- to brain immediately.

#### 3.2. Output of control $u[t]$ :

- common (by means of hands, feet and voice; by top of head in diving suits);
- by reading nerve impulses in hands [2];
- from brain immediately.

General conclusion from [2]: using appropriate equipment for feedback, each physiological display (breath, pulse etc.) by human or animal with cognitive ability (ape, dolphin, dog) can be used for control.



3.3. There is Galileo-Einstein's principle of relativity: if we observe uniform movement of an object towards us then we cannot detect whether the object is moving, or we are.

For virtual motion the condition of uniformity (i.e. no acceleration) is not necessary. Hence, we receive a principle of relativity in virtual motion: if we observe movement in a kinematical space [3] then we can interpret it either movement of space toward us or our movement toward space.



## 4. Some cases of computer-human control

4.1. Simulation cases of real control where real training is too difficult, expensive or dangerous include: spaceship, aircraft, boat, U-boat, artillery, launching big rockets, manufacturing processes, medical operations. They consist of random generation of media and random generation of emergencies. Simulation made for a crew (for instance, pilot, co-pilot and navigator at aircraft) is an example of necessary-collective activity.

**Remark.** Some simulators are mixed computer-mechanical solutions that involve vibration and physical inclinations.

## 4.2. Computer games. Notes:

- some computer games arose from items listed in 4.1;
- computer games involving simulations of real objects (geographical map, concrete vehicles) may be considered educational;
- there are some hints in computer games useful to forthcoming proposals.

**Remark.** We do not consider games “person versus person” and “team versus team” by means of computers.

4.3. Imitation of physical-chemical experiments - “virtual laboratory”.

4.4. Enhancing virtual reality. We [3] proposed to present abstract spaces in form close to presentation of the metric space.

**Definition.** A pair: a set  $X$  of points and a set  $K$  of **routes** is said to be a **kinematic space** (each route  $M$ , in turn, consists of the positive real number  $T_M$  (**time** of route) and the function  $m_M: [0, T_M] \rightarrow X$  (**trajectory** of route)) if the following conditions are fulfilled:


(K1) For  $x_0 \neq x_1 \in X$  there exists such  $M \in K$  that  $m_M(0) = x_0$  and  $m_M(T_M) = x_1$ , and the set of values of such  $T_M$  is bounded with a positive number below (infinitely fast motion is impossible);

(K2) If  $M = \{T_M, m_M(t)\} \in K$  then the pair  $\{T_M, m_M(T_M - t)\}$  is also a route of  $K$  (the reverse motion is possible);

(K3) If  $M = \{T_M, m_M(t)\} \in K$  and  $T^* \in (0, T_M)$  then the pair:  $T^*$  and function  $m^*(t) = m_M(t)$  ( $0 \leq t \leq T^*$ ) is also a route of  $K$  (one can stop at any desired moment);

(K4) concatenation.

We implemented controlled motion in Riemann surfaces, Moebius band, projective plane and topological torus.



4.5. Experimental mathematics [4]. On one hand, it is using well-known software (Mathematica, MathLab, MathCad), on the other hand a search for new mathematical facts (hypothesis) - a separate direction of investigations.

4.6. Training in deciphering the simplest ciphers alongside with evaluating the knowledge of a language [5].

4.7. Complex examination (for example, [7]) including multimedia tasks, interactive tasks of optimization and solving equations, tasks with objects with-out. Primary versions of such software for Kyrgyz language, mathematics and informatics were implemented and are in use.

4.8. Measuring imagery [8]. **Definition.** The problem is said to be intellectual eye measurer (or measuring imagery) - its conditions are strict but the approximate answer is permissible; using any tool (computer, pen-and-paper, reference book) is forbidden; in sciences the time given to answer is about 20 - 30 seconds to avoid immediate mental counting. If the answer is a real number then  $Q(x) = |x - x_0|$  or  $Q(x) = |\log(x/x_0)|$  (for  $x_0 > 0$ ) where  $x_0$  is the exact answer.

We have introduced competitions on students' capacities in this subject matter.

4.9. Necessarily-collective tasks [9]. For example such task includes: transformation of sign systems: the first teammate is given a drawing (a set of similar drawings); s/he describes it in a prescribed language (during 15-20 minutes) and this text is sent to the second teammate by an intermediary; s/he restores the drawing (the consequence of drawings) (during 10-15 minutes).

4.10. Software to correct pronunciation.

4.11. Independent interactive presentation of objects. If a computer presentation does not depend on the user's knowledge and skills on similar objects then it is said to be independent.

4.11a. Interactive presentation of some mathematical objects [10].

4.11b. Interactive presentation of basic of language. Earlier, learning a living language was implemented with the assistance (including bilingual dictionaries and text-books) of persons who had a complete command of it; investigating of a dead language was done by means of remained bilingual texts and texts with additional implicit suggestions and conclusions. Invention of recording sounds gave possibility to fix examples of an oral language objectively. Invention of talking pictures fixed examples of phrases with connection to situations and actions. Computer games gave the user the opportunity to choose actions with corresponding phrases.

Before our publications, existed software to learn languages were based on languages native to the user.

We proposed [11-15] **Definition.** Let any "notion" (word of a language) be given. If an algorithm acting at a computer: - performs (generating randomly) sufficiently large amount of situations covering all essential aspects of the "notion" to the user; - gives a command involving this "notion" in each situation; - perceives the user's actions and performs their results clearly on a display; - detects whether a result fits the command, then such algorithm is said to be a computer interactive presentation of the "notion".

Simple mathematical models consist of fixed ( $F_i$ ) and movable ( $M_j$ ) sets and temporal sequence of conditions of types ( $M_j \subset F_i$ ), ( $M_j \cap F_i = \emptyset$ ), ( $M_j \cap F_i \neq \emptyset$ ).

**Remark.** 4.10) can also be involved.

Sketches of such software were implemented for Kyrgyz, English and Turkish languages. A proposal for Chinese language was in [16].

## 5. Conclusion

We hope that developing this method would yield new types of educational software both interesting and useful for students. For instance, combination of 4.3) and 4.11a) can give independent presentation of some physical notions; adding of mathematical tasks with physical content can give a complex examination in physics.



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**Thank you for attention!**